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A STATISTICAL ANALYSIS OF NUCLEATE-POOL-BOILING DATA

G. A. HUGHMARK
Ethyl Corporation, Baton Rouge, Louisiana

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Abstract—Nucleate pool boiling experimental data have been analyzed with a statistical technique to obtain an eight variable equation for the heat flux. The dimensional derived equation considers only the thermodynamic properties of the liquid and vapour and neglects the effects of the surface condition, the geometry and orientation. The equation has no theoretical justification but it does indicate the best agreement with the experimental data that can be obtained when only the thermodynamic properties are considered as products of powers. An average absolute deviation of 40 per cent is obtained between experimental and predicted heat flux for a range of materials from boiling hydrogen to boiling mercury.

NOMENCLATURE

- C, specific heat, Btu/lb °F;
- k, thermal conductivity, Btu/h ft^2 °F per ft;
- L, latent heat of vaporization, Btu/lb;
- P, absolute pressure, lbf/ft^2;
- $\bar{q}$, heat flux, Btu/h ft^2;
- T, absolute temperature, °R;
- ΔT, $T_W - T_S$, °F;
- ΔP, $P_W - P_S$, lbf/ft^2;
- μ, viscosity, lb/ft h;
- ρ, density, lb/ft^3;
- σ, surface tension, lbf/ft.

Subscripts

- L, liquid;
- R, reduced temperature or pressure;
- S, at saturation temperature;
- V, vapor;
- W, at wall temperature.

A WEALTH of experimental data has been published for the nucleate pool boiling of liquids. Many equations have been proposed to predict the heat flux as obtained from these experimental data, but no general equation has been developed.

Several recent publications [1–3] have proposed correlations for heat flux based on the system's physical properties, the temperature difference between the heated surface and the saturation temperature, and the pressure differential corresponding to the superheat. These correlations are based upon dimensionless groups derived from theory. The correlations are in fair agreement with some of the data but are several hundred per cent out from other experimental data.

Other recent papers have included the study of the heating surface as a variable [4, 5]. A large amount of work is in progress to determine the effect of the nature of the heat-transfer surface on the flux.

The object of the work covered by this paper is to apply a statistical analysis to obtain an equation for heat flux with a minimum deviation from the published experimental data. This analysis considers only the thermodynamic properties of the liquid and vapor and neglects the effects of the surface condition, the geometry and orientation. Such an analysis is a brute-force technique and is devoid of physical interpretation, but it is useful when a large amount of experimental data are available, because it does provide:

(1) the optimum result if only the thermodynamic properties are considered as product of powers;
(2) an estimate of the maximum and the average deviation which can be expected between predicted and experimental results.

A statistical method of analysis is reported for the related problem of burnout heat flux for water in circular pipe [6]. A least-squares technique was used to fit the experimental data with four independent variables and a sixteen-constant polynomial equation.

STATISTICAL ANALYSIS

The heat flux for nucleate pool boiling can be considered as a variable, dependent on many independent variables. The independent variables can be divided into several different categories:

(1) Physical properties of the liquid and vapor at the temperature of the heated surface and at the saturation temperature.
(2) Temperature and pressure of the system.
(3) Temperature difference between the heated surface and the saturation temperature.
(4) Pressure difference corresponding to the superheat of the heated surface.
(5) Physical characteristics of the surface.

Categories (1–4) are readily defined for a system. The characteristics of Category (5) are difficult to determine, and surface measurements are included for only a limited amount of the experimental data. Therefore, this analysis is applied to only the first four categories and shows what can be considered as the best agreement for a correlation including these variables.

The variables can then be written as follows:

	At wall temp.	At saturation temp.
Category (1)	$(\rho_L)_W$	$(\rho_L)_S$
	$(\rho_V)_W$	$(\rho_V)_S$
	$(\rho_L - \rho_V)_W$	$(\rho_L - \rho_V)_S$
	$(\mu_L)_W$	$(\mu_L)_S$
	σ_W	σ_S
	$(\mu_V)_W$	$(\mu_V)_S$
	$(C_V)_W$	$(C_V)_S$
	$(k_V)_W$	$(k_V)_S$
	$(C_L)_W$	$(C_L)_S$
	$(k_L)_W$	$(k_L)_S$
	L_W	L_S
Category (2)	T_W, T_S, T_R based on T_W and T_S	
	P_W, P_S, P_R based on P_W and P_S	
Category (3)	$\Delta T = T_W - T_S$	
Category (4)	$\Delta P = P_W - P_S$	

The total number of variables is thus determined to be thirty-two. A multivariate correlation program available for the LGP-30 computer is limited to fourteen independent variables. An elimination process was used as follows to reduce the number of variables.

(a) As nucleate-boiling heat transfer is considered to be transfer from a hot surface to liquid, elimination of vapor viscosity, vapor specific heat and vapor thermal conductivity appears justified. This reduces the variables by six.
(b) Liquid density, liquid density minus vapor density, specific heat, latent heat and absolute temperature do not change appreciably from T_W to T_S. Eliminating these variables at T_S reduces the variables by six.
(c) Reduced temperature and pressure also do not vary appreciably from T_W to T_S and P_W to P_S. Arbitrarily eliminating T_R and P_R at T_W and P_W reduces the variables by two to a total of eighteen remaining.
(d) Multivariate correlation with a portion of the data indicates that T_R, ΔT, P_W and P_S are not significant variables. Elimination of these reduces the number of variables to the following fourteen:

$(\rho_L)_W$	L_W
$(\rho_V)_W$	$(\rho_V)_S$
$(\rho_L - \rho_V)_W$	$(\mu_L)_S$
$(\mu_L)_W$	σ_S
σ_W	T_W
$(C_L)_W$	ΔP
$(k_L)_W$	P_R

The fourteen variables were used as the independent variables and heat flux as the dependent variable. Heat flux data from the literature, as shown by Table 1, were plotted versus ΔT. A line was drawn through the data for a constant system pressure. The end points and an intermediate point were taken from each of these lines and were used as data points for the correlation. A total of 340 points was obtained

Table 1. Nucleate-pool-boiling data

Material	Pressure (lb/in^2)*	Reference
Water	1·28–14·7	[7]
Water	14·7–2465	[8]
Water	14·7	[9]
Water	14·7–1015	[10]
Water	40 mm Hg–33 lb/in^2	[11]
Water	14·7	[12]
Water	14·7	[13]
Water	14·7	[14]
Water	0·54–22·4	[15]
Water	2·28–14·7	[16]
Water	14·7–1425	[17]
Methanol	1·25–20	[15]
Methanol	14·7	[18]
Ethanol	14·7	[12]
Ethanol	14·7–115	[10]
Ethanol	15–265	[19]
Isopropanol	200 mm Hg–14·7 lb/in^2	[16]
Isopropanol	14·7	[6]
n-Butanol	200 mm Hg–14·7 lb/in^2	[16]
n-Butanol	14·7	[12]
n-Butanol	125 mm Hg–24 lb/in^2	[15]
Benzene	14·7–265	[10]
Benzene	15–215	[19]
Benzene	14·7	[20]
Propane	170–475	[10]
Propane	80	[21]
n-Butane	23	[21]
n-Pentane	14·7–415	[10]
n-Pentane	14·7	[22]
n-Heptane	235 mm Hg–33 lb/in^2	[11]
n-Heptane	6·6–215	[10]
n-Heptane	14·7	[13]
Styrene	32–450 mm Hg	[11]
Butadiene	60–90	[11]
Acetone	14·7	[12]
Acetone	15–115	[19]
Methyl ethyl ketone	14·7	[18]
Diethyl ether	14·7	[22]
Methyl chloride	48–60	[21]
Carbon tetrachloride	3–19	[15]
Freon 12	48·1–71·2	[21]
Freon 113	15–40	[19]
Sulfur dioxide	29–46·5	[25]
Mercury	14·7–44	[23]
Mercury	14·7	[24]
Oxygen	14·7	[25]
Nitrogen	14·7	[25]
Hydrogen	14·7	[26]

* Unless otherwise stated.

in this manner. A least-squares calculation was made with the LGP-30 computer to obtain the best fit to the equation.

$$\log \bar{q} = \log a_0 + a_1 \log \Delta P + a_2 \log (\rho_L)_W + a_3 \log (\rho_V)_W + a_4 \log (\rho_L - \rho_V)_W + a_5 \log (k_L)_W + a_6 \log (C_L)_W + a_7 \log L_W + a_8 \log \sigma_W + a_9 \log (\mu_L)_W + a_{10} \log T_W + a_{11} \log \sigma_S + a_{12} \log (\mu_L)_S + a_{13} \log (\rho_V)_S + a_{14} \log P_R.$$

The Pearson correlation coefficient for the least-squares fit was 0·925, which indicates a high level of significance of the fit of the equation to the data.

Standard deviations for the coefficients of each of the variables were obtained as part of the computer program. The coefficients for $(\rho_L)_W$, $(k_L)_W$, σ_W, σ_S, $(\mu_L)_S$ and $(\rho_V)_S$ were found to be less than twice the standard deviation for the coefficient. This indicates that these variables are not statistically significant. A calculation was then repeated without these six variables. The Pearson correlation coefficient was found to be 0·923 with the eight remaining variables. Coefficients for the remaining variables were all significant at the two-standard deviation level. Calculations were repeated for each combination of seven of the remaining eight variables. In each case, the correlation coefficient decreased appreciably which confirmed that the eight remaining variables are significant.

The resulting equation is then

$$\bar{q} = 2{\cdot}67 \times 10^{-9} \frac{\Delta P^{1\cdot867} (\rho_L - \rho_V)_W{}^{2\cdot27} (C_L)_W{}^{0\cdot945} T_W{}^{1\cdot618}}{(\rho_V)_W{}^{1\cdot385} L_W{}^{1\cdot15} (\mu_L)_W{}^{1\cdot630} P_R{}^{0\cdot202}}.$$

The data points are plotted as Fig. 1. The average absolute deviation of the predicted values from the experimental values is 40 per cent.

The experimental data were correlated with several combinations of the significant variables into dimensionless groups. In all groups the average absolute deviation was considerably greater then the 40 per cent obtained for the dimensional equation.

Although correlation with part of the data indicated that T_R, ΔT, P_W and P_S were not significant variables, these four variables were

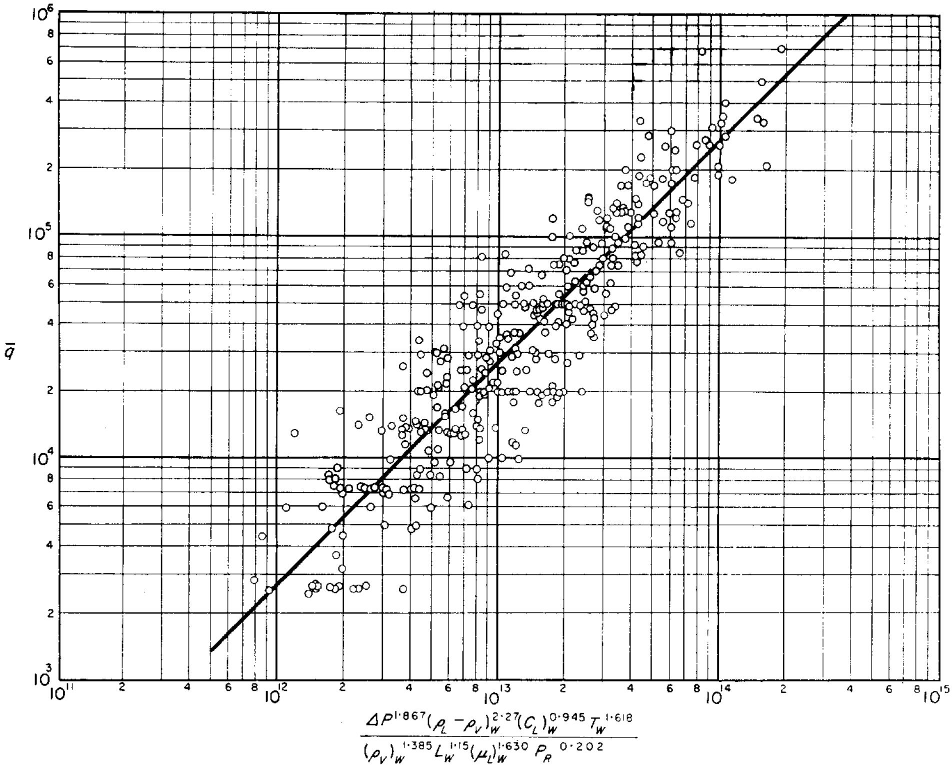

FIG. 1. Correlation for heat flux with nucleate pool boiling.

combined with the eight significant variables from analysis of all of the data and a least-squares fit was obtained for all the data and the twelve variables. The correlation coefficient was only slightly different from that obtained with the eight variables.

Table 2 shows the absolute deviation between predicted and experimental heat fluxes for several systems averaged for the three points of the smoothed data. The deviations between experimental and predicted values are also shown for the methods of Forster–Greif, Levy and Nishikawa–Yamagata.

SUMMARY

A statistical technique has been used to obtain an equation for heat flux in nucleate pool boiling. An eight-variable equation was found to describe the heat flux with an average absolute deviation of 40 per cent by comparison with the experimental data reported in the literature.

The derived equation has no theoretical justification; in fact it does not contain liquid thermal conductivity which is essential to theoretical derivations. The 40 per cent average absolute deviation does show the best agreement that can be obtained with the experimental data when only thermodynamic properties are considered as products of powers. Consideration of other variables, such as surface effects, appears to be necessary to obtain a better fit with the experimental data.

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Table 2. Average absolute deviations between predicted and experimental heat fluxes

Material	Pressure (lb/in^2)	Data reference	Per cent average absolute deviations Method [1]	Method [2]*	Method [3]	This paper
Water	2·28	16	33	61	87	35
Water	6·42	16	31	67	71	13
Water	14·7	16	32	61	40	12
Ethanol	14·7	10	163	46	60	36
Ethanol	55	10	24	27	65	34
Ethanol	115	10	28	16	55	44
n-Pentane	415	10	30	76	96	15
Propane	170	10	210	70	15	19
Propane	475	10	450	390	178	13
Mercury	14·7	23	27	90	13 000	23
Nitrogen	14·7	25	190	49	93	32

* It is assumed that $1/B_L$ is multiplied by 10^{-5}, rather than 10^{-6} as shown by the reference.

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Résumé—L'étude statistique des données expérimentales sur l'ébullition nucléée a permis d'établir une équation de flux thermique à 8 variables. L'équation dimensionelle ne tient compte que des propriétés thermodynamiques du liquide et de la vapeur, les effets de la condition de surface, de la géométrie et de l'orientation sont négligés. L'équation n'a pas de justification théorique mais est vérifiée au mieux par les données expérimentales quand on considère uniquement les propriétés thermodynamiques comme produits de puissance. La divergence moyenne absolue entre les flux déterminés par l'expérience et le calcul est de 40% pour une série de corps allant de l'hydrogène au mercure.

Zusammenfassung—Nach einer statistischen Methode wurden Versuchsergebnisse der Blasenverdampfung bei freier Konvektion analysiert und eine Gleichung für den Wärmefluss mit 8 Variablen ermittelt. Die erhaltene Beziehung berücksichtigt nur die thermodynamischen Eigenschaften der Flüssigkeit und des Dampfes und vernachlässigt Oberflächenbedingung, Geometrie und Orientierung. Die Gleichung lässt sich nicht theoretisch rechtfertigen, vermittelt aber die bestmögliche Übereinstimmung mit Versuchswerten soweit die thermodynamischen Eigenschaften als Potenzprodukte betrachtet werden. Für einen Bereich siedender Medien von Wasserstoff bis Quecksilber ergibt sich als mittlere absolute Abweichung zwischen theoretisch und experimentell ermitteltem Wärmefluss ein Wert von 40%.

Аннотация—С помощью статистических приёмов исследованы экспериментальные данные по пузырьковому кипению и получено уравнение теплового потока с восьмью переменными. Выведенное размерное уравнение учитывает только термодинамические свойства жидкости и пара. Не учтено влияние условий на поверхности, её геометрии и расположения. Уравнение теоретически не обосновано, но даёт наилучшее соответствие с экспериментальными данными. Средняя абсолютная погрешность между экспериментальными и расчётными значениями тепловых потоков равна 40% для различных веществ (от кипящего водорода до кипящей ртути).